Matrices

37

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 3 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$, and $\vec{b} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$.

37.1 Compute the product $A\vec{x}$.

37.2 Write down a system of equations that corresponds to the matrix equation $A\vec{x} = \vec{b}$.

37.3 Let $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$ be a solution to $A\vec{x} = \vec{b}$. Explain what x_0 and y_0 mean in terms of *intersecting lines* (hint: think about systems of equations).

37.4 Let $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$ be a solution to $A\vec{x} = \vec{b}$. Explain what x_0 and y_0 mean in terms of *linear combinations* (hint: think about the columns of A).

38 Let $\vec{u}=\begin{bmatrix}1\\2\\3\end{bmatrix}$, $\vec{v}=\begin{bmatrix}4\\5\\6\end{bmatrix}$, $\vec{w}=\begin{bmatrix}7\\8\\9\end{bmatrix}$.

38.1 How could you determine if $\{\vec{u},\vec{v},\vec{w}\}$ was a linearly independent set?

38.2 Can your method be rephrased in terms of a matrix equation? Explain.

39 Consider the system represented by

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \vec{b}.$$

39.1 If $\vec{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, is the set of solutions to this system a point, line, plane, or other?

39.2 If $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, is the set of solutions to this system a point, line, plane, or other?

40 Let $\vec{d}_1 = \begin{bmatrix}1\\1\\2\end{bmatrix}$ and $\vec{d}_2 = \begin{bmatrix}-1\\1\\0\end{bmatrix}$. Let $\mathcal{P}$ be the plane given in vector form by $\vec{x} = t\vec{d}_1 + s\vec{d}_2$. Further, suppose M is a matrix so that $M\vec{r} \in \mathcal{P}$ for any $\vec{r} \in \mathbb{R}^2$.

40.1 How many rows does M have?

40.2 Find such an M.

40.3 Find necessary and sufficient conditions (phrased as equations) for $\vec{n}$ to be a normal vector for $\mathcal{P}$.

40.4 Find a matrix K so that non-zero solutions to $K\vec{x} = \vec{0}$ are normal vectors for $\mathcal{P}$. How do K and M relate?